

**THE ACCURACY OF THE HAMILTON-PERRY METHOD FOR  
FORECASTING STATE POPULATIONS BY AGE: A LONG TERM  
TEST USING FOUR RANDOMLY SELECTED U.S. STATES**

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# Overview

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## Overview

**The Hamilton-Perry Method is a variant of the Cohort-Component population projection method that has minimal data input requirements. It only requires the age distributions for a population at two points in time, which generally are two successive census enumerations. Although the method has gained acceptance, tests of its accuracy are limited. In this paper we evaluate the forecast accuracy of the Hamilton-Perry method both in terms of age and total population (the latter obtained by summing up the forecasted age groups).**

## Overview

**This evaluation is based using a sample of four states (one from each of the four census regions) and decennial census data from 1900 to 2010, which yield 10 census test points (1920, 1930, 1940, ..., 2010) or target years, which provide a wide range of characteristics in regard to population size, growth, and age-composition, factors that affect forecast accuracy. We conclude that the results are encouraging and suggest that the Hamilton-Perry Method be considered when either a 10-year forecast of a state population by age or a total population are desired and components of change are not required.**

## Background

Although the Hamilton-Perry Method has primarily been used for small geographic areas, its minimal data input requirements combined with its capability for producing age and other characteristics in a forecast make it attractive for use at high levels of geography such as states and counties. To our knowledge only one study, by Smith and Tayman (2003), has evaluated the accuracy of the Hamilton-Perry method. They examined forecast errors by age group for all states and for counties in Florida using 1990 and 2000 as target years and ten- and 20-year forecast horizons. They found that its accuracy was equivalent to that of cohort-component method forecasts. This paper is

## Background

**This paper is intended to supplement their findings by using a sample of states in conjunction with tests covering a long period of time. Unlike Smith and Tayman (2003), we do not evaluate all states, only a sample of four (one randomly selected from each of the four census regions, respectively); but unlike them we use not just two recent target years, but nine target years, from 1920 to 2010 that represent 10-year forecast horizons.**

# Background

**In the discussion that follows, we begin by describing the Hamilton-Perry Method and then move on to discuss three major dimensions of forecast accuracy and how they are measured. We then describe the data used in our empirical examination and our results, which include an evaluation of the accuracy of the projected age groups and the projected total populations. We conclude the paper with a discussion of our findings, their implications and limitations, which, in turn, lead to suggestions for future research in this area.**

## Hamilton-Perry Method

Before describing the Hamilton-Perry method, it is useful to recall that any quantitative approach to forecasting is constrained to satisfy various mathematical identities (Land 1986). In regard to population forecasting, an approach should ideally satisfy demographic accounting identities, which is summarized in the identity known as the fundamental demographic equation:

$$P_t = P_0 + \text{Births} - \text{Deaths} + \text{Inmigrants} - \text{Outmigrants}. \quad [1]$$



## Hamilton-Perry Method

That is, the population at some time in the future,  $P_t$ , must be equal to the population at an earlier time,  $P_0$ , plus the births and in-migrants and less the deaths and out-migrants that occur between time 0 and time  $t$ . The most commonly used approach to population forecasting, the cohort-component method satisfies the fundamental equation, but it is data-intensive.

## Hamilton-Perry Method

**In Appendix 1 of our full paper, we show that the Hamilton-Perry Method satisfies the fundamental demographic equation. Moreover, it has far less intensive input data requirements than does the cohort-component method.**

**Instead of mortality, fertility, migration, and population data by age and sex, which are required by the full-blown cohort-component method, the Hamilton-Perry method requires age data only from the two most recent censuses.**

## Hamilton-Perry Method

**However, like the full-blown cohort-component method, the Hamilton-Perry Method can be used to develop projections not only by age, but also by age and sex, age and race, age, sex and race, and so on.**

## Hamilton-Perry Method

The Hamilton-Perry method projects a population by age (and sex) from time (t) to time (t+k) using “COHORT CHANGE RATIOS” (CCRs) computed from the two most recent censuses. It consists of two steps. The first uses existing data to develop CCRs and the second applies the CCRs to the cohorts of the launch year population to move them into the future. The second step can be repeated infinitely, with the projected population serving as the launch population for the next projection cycle.

## Hamilton-Perry Method

What are Cohort Change Ratios (CCRs)? They have a long history of use in demography. Under the rubric of “Census Survival Ratios,” they have been used to estimate adult mortality and under the rubric of the “Hamilton-Perry” method, to make population projections.

## Hamilton-Perry Method

The formula for the first step, the development of a CCR is:

$${}_n\text{CCR}_x = {}_n\text{P}_{x,t} / {}_n\text{P}_{x-k,t-k}$$

where

${}_n\text{P}_{x,t}$  is the population aged  $x$  to  $x+n$  at the most recent census ( $t$ ),

${}_n\text{P}_{x-k,t-k}$  is the population aged  $x-k$  to  $x-k+n$  at the 2nd most recent census ( $t-k$ ),

$k$  is the number of years between the most recent census at time  $t$  and the one preceding it at time  $t-k$ .

# Hamilton-Perry Method

The basic formula for the second step, projecting age cohorts is:

$${}_n P_{x+k,t+k} = ({}_n CCR_x) \times ({}_n P_{x,t})$$

where

${}_n P_{x+k,t+k}$  is the population aged  $x+k$  to  $x+k+n$  at time  $(t+k)$ , and

${}_n P_{x,t}$  is the population aged  $x$  to  $x+n$  at the most recent census  $(t)$ .

## Hamilton-Perry Method

Given the nature of the CCRs, 10-14 is the youngest age group for which projections can be made if there are 10 years between censuses. To project the populations aged 0-4 and 5-9, one can use the Child Woman Ratio (CWR), or more generally a “Child Adult Ratio” (CAR). It does not require any data beyond the decennial census. For projecting the population aged 0-4, CAR is defined as the population aged 0-4 divided by the population aged 15-44. For projecting the population aged 5-9, CAR is defined as the population aged 5-9 divided by the population aged 20-49.\*

\*There are other “adult” age groups that could be used to define CAR.



## Hamilton-Perry Method

Another way to obtain “CCRs” for the two youngest age groups is to take their ratios at two points in time and apply that ratio to the launch year age group ( $t$ ). In the first step, the ratios are :

$$\text{Population 0-4: } {}_5R_{0,t} = {}_5P_{0,t} / {}_5P_{0,t-k}$$

$$\text{Population 5-9: } {}_5R_{5,t} = {}_5P_{5,t} / {}_5P_{5,t-k}$$

In the second step, the projected population at  $t+k$  is found by:

$$\text{Population 0-4: } {}_5P_{0,t+k} = {}_5P_{0,t} \times {}_5R_{0,t}$$

$$\text{Population 5-9: } {}_5P_{5,t+k} = {}_5P_{5,t} \times {}_5R_{5,t}$$

## Hamilton-Perry Method

We use the former method since it is more common. However, we note that since the average age at first birth has increased over the period for which our test data were assembled, the constant set of CARs we use are not optimal for many of the test points.

## THREE DIMENSIONS OF FORECAST ACCURACY

There are three major dimensions to forecast (and estimation) accuracy: (1) bias; (2) precision; and (3) allocation (Swanson et al., 2012). There are three measures that correspond to each of these three dimensions, respectively (Swanson, et al., 2012): (1) Mean Algebraic Percent Error (MALPE); (2) Mean Absolute Percent Error (MAPE); and (3) The Index of Allocation Error (IOAE).

## THREE DIMENSIONS OF FORECAST ACCURACY

In moving toward definitions of these three measures, we begin by defining Forecast error (E), which is the difference between a given forecast (F) for a particular population and the 2010 census (CEN):  $E = F - \text{CEN}$ . The error will be positive when the forecast is larger than the census count and it will be negative when it “under-forecasts” the census. The definition of error given above can be broadened to include age, sex, race, and ethnicity. For example, the definition of error for a particular age group (a) would be  $E_a = F_a - \text{CEN}_a$ .

## THREE DIMENSIONS OF FORECAST ACCURACY

Based on previous research and our experience (e.g., Smith and Tayman 2003; Smith, Tayman, and Swanson 2001: 339-340; Swanson and Beck 1994; Swanson et al., 2012), we believe that for a ten-year forecast of total population for states, a MALPE less than +5% and greater than -5% indicates that a forecast is not substantially biased. A MALPE greater than +5% but less than +15% or less than -5% but not less than -15% indicates considerable bias and one greater than +15% indicates substantial upward bias (the forecast is way too high) and one less than -15% indicates substantial downward bias (the forecast is way too low)

## THREE DIMENSIONS OF FORECAST ACCURACY

A MAPE less than 5% indicates that the forecast is highly precise, while a MAPE greater than 5% but less than 10% indicates a modest level precision, and, finally, a MAPE greater than 10% indicates a low level of precision.

## THREE DIMENSIONS OF FORECAST ACCURACY

The summary measures discussed above are based on the error for a particular geographic area. Another perspective views the misallocation of the forecast across geographic space or a given variable such as age. Misallocation focuses on how well the forecast distribution matches the observed distribution such as the total population distribution across states or the age distribution in a particular state.

## THREE DIMENSIONS OF FORECAST ACCURACY

The Index of Allocation Error (IOAE), also known as the Index of Dissimilarity) can be used to measure the extent that the forecast misallocates a given variable, such as age. IOAE compares the percent distributions of the forecast and census shares across the categories of a given variable (e.g., age) and measures the percentage that one distribution (i.e. based on the forecasts) would have to be re-allocated to match the other (i.e. based on the census).



## THREE DIMENSIONS OF FORECAST ACCURACY

The IOAE ranges from 0 to 100; a score of zero is means that there is no allocation error, and 100 means that the maximum allocation error exists. This can mean several things, but a common interpretation is that half of the forecast numbers would have to be re-allocated and half of the census counts would have to be re-allocated.

## THREE DIMENSIONS OF FORECAST ACCURACY

Again, based on previous research and our experience (e.g., Swanson and Beck 1994; Swanson et al. 2012), we believe that an IOAE of less than five percent indicates a very close match between the two distributions, while an IOAE between five and ten percent portrays a reasonable match between the two distributions, and an IOAE greater than ten percent suggests the distributions are quite distinct.

## TEST DATA

To empirically examine the accuracy of the Hamilton-Perry Method, we selected a sample made up of one state from each of the four census regions in the United States. Within each Census Region, a state was randomly selected. The states selected are Georgia (the South Region), Minnesota (the Midwest Region), New Jersey (The Northeast Region) and Washington (The West Region). We then assembled census data for these four states for each census year from 1900 to 2010.

## TEST DATA

The data provide ten points in time at which the forecast intervals can be evaluated, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000, and 2010. This sample provides a wide range of demographic characteristics in terms of variation in population size, age-composition, and rates of change

## TEST DATA

Table 1 provides an overview of this range by displaying the population of each of the four states in 1900 and in 2010 and decennial rates of population change from 1900 to 2010. Although we do not show a summary of the changes in age composition by state and census year, they are extensive as seen in Appendix 2, which provide the age data by state and census year.

# TEST DATA

Table 1. Total Population 1900 and 2010 and Annual Rate of Change by Decade, Sample States

| Census Year       | Georgia   | Minnesota | New Jersey | Washington |
|-------------------|-----------|-----------|------------|------------|
| 1900 <sup>a</sup> | 2,209,974 | 1,747,292 | 1,879,890  | 511,844    |
| 1900-1910         | 1.64%     | 1.70%     | 2.99%      | 7.97%      |
| 1910-1920         | 1.05%     | 1.41%     | 2.19%      | 1.75%      |
| 1920-1930         | 0.05%     | 0.72%     | 2.47%      | 1.44%      |
| 1930-1940         | 0.72%     | 0.86%     | 0.30%      | 1.06%      |
| 1940-1950         | 0.98%     | 0.66%     | 1.50%      | 3.14%      |
| 1950-1960         | 1.35%     | 1.35%     | 2.27%      | 1.83%      |
| 1960-1970         | 1.52%     | 1.08%     | 1.67%      | 1.78%      |
| 1970-1980         | 1.74%     | 0.69%     | 0.27%      | 1.92%      |
| 1980-1990         | 1.70%     | 0.71%     | 0.48%      | 1.64%      |
| 1990-2000         | 2.34%     | 1.17%     | 0.85%      | 1.92%      |
| 2000-2010         | 1.68%     | 0.75%     | 0.44%      | 1.32%      |
| 2010              | 9,687,653 | 5,303,925 | 8,791,894  | 6,724,540  |

<sup>a</sup> The 1900 population totals exclude those for whom age was not reported.

## TEST DATA

Because of the way data for the terminal open-ended age group are reported differently over the period for which we assemble census data, we used "75 years and over" for the entire period since it was the common denominator. This means that there are 16 age groups (0-4, 5-9, ..., 70-74, and 75+) used in the empirical evaluation.

## RESULTS

We begin the analysis by discussing the errors across all age groups by state and test target year (see Table 2) and the errors for the total population forecasts by state and test target year (See Table 3). Keep in mind that the total population forecasts in Table 3 are found by summing the forecasts for each age group. The Hamilton-Perry Method does not produce require a “direct” forecast of a total population, although some applications control the Hamilton-Perry projections to an independent projection of total population. We opted not to control the Hamilton-Perry projections by age because we wanted to evaluate the total population forecast error derived directly from the method itself.



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## RESULTS

Table 2. Forecast Errors by State and Target Year<sup>a</sup>

| Target Year         | Georgia            |                   |                   | Minnesota          |                   |                   | New Jersey         |                   |                   | Washington         |                   |                   |
|---------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|--------------------|-------------------|-------------------|
|                     | MALPE <sup>b</sup> | MAPE <sup>c</sup> | IOAE <sup>d</sup> | MALPE <sup>b</sup> | MAPE <sup>c</sup> | IOAE <sup>d</sup> | MALPE <sup>b</sup> | MAPE <sup>c</sup> | IOAE <sup>d</sup> | MALPE <sup>b</sup> | MAPE <sup>c</sup> | IOAE <sup>d</sup> |
| 1920                | 3.65               | 5.04              | 2.27              | 2.42               | 4.78              | 3.00              | 7.30               | 7.30              | 2.43              | 77.15              | 77.15             | 7.03              |
| 1930                | 8.12               | 8.22              | 3.11              | 2.74               | 4.46              | 2.46              | -6.06              | 8.25              | 2.46              | -0.76              | 4.96              | 2.77              |
| 1940                | -8.39              | 11.28             | 4.29              | -3.60              | 6.35              | 2.40              | 21.37              | 21.37             | 5.88              | 2.15               | 5.59              | 3.23              |
| 1950                | -4.13              | 6.16              | 3.50              | 2.06               | 6.44              | 3.91              | -11.55             | 11.55             | 4.04              | -16.49             | 16.49             | 6.09              |
| 1960                | -6.36              | 7.09              | 3.84              | -7.42              | 7.80              | 6.13              | -9.28              | 9.28              | 6.27              | 11.43              | 14.86             | 5.85              |
| 1970                | -3.41              | 8.29              | 3.59              | 0.11               | 6.30              | 4.16              | 3.56               | 6.40              | 3.74              | -0.19              | 7.39              | 4.37              |
| 1980                | -0.75              | 10.86             | 6.30              | 7.78               | 0.10              | 8.57              | 19.62              | 18.97             | 8.47              | 2.50               | 11.38             | 7.19              |
| 1990                | 0.19               | 3.29              | 1.80              | 0.50               | 2.01              | 1.19              | -1.36              | 4.14              | 1.82              | 1.93               | 3.96              | 1.61              |
| 2000                | -7.94              | 7.94              | 2.39              | -7.57              | 7.57              | 1.62              | -6.01              | 6.01              | 2.63              | -5.93              | 5.93              | 2.12              |
| 2010                | 4.29               | 5.64              | 2.01              | 2.98               | 3.58              | 1.28              | 1.59               | 2.95              | 1.45              | 4.76               | 5.65              | 1.63              |
| Mean                | -1.47              | 7.38              | 3.31              | 0.00               | 4.94              | 3.47              | 1.92               | 9.62              | 3.92              | 7.66               | 15.34             | 4.19              |
| Std. Deviation      | 5.57               | 2.49              | 1.34              | 4.86               | 2.48              | 2.35              | 11.39              | 6.10              | 2.28              | 25.48              | 22.15             | 2.21              |
| Coeff. of Variation | -3.78              | 0.34              | 0.41              | N/A                | 0.50              | 0.68              | 5.94               | 0.63              | 0.58              | 3.33               | 1.44              | 0.53              |
| Median              | -2.08              | 7.52              | 3.31              | 1.28               | 5.54              | 2.73              | 0.12               | 7.78              | 3.19              | 2.04               | 6.66              | 3.80              |

<sup>a</sup> Summary of errors for all age groups.

<sup>b</sup> MALPE = Mean Algebraic Percent Error.

<sup>c</sup> MAPE = Mean Absolute Percent Error.

<sup>d</sup> IOAE = Index of Allocation Error (Also known as the Index of Dissimilarity).

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## RESULTS

Table 3. Forecast Errors for Total Population by State and Target Year<sup>a</sup>

| Target Year         | Georgia         |                  | Minnesota       |                  | New Jersey      |                  | Washington      |                  | APE (All States)                  |              |      |        |
|---------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|-----------------------------------|--------------|------|--------|
|                     | Pe <sup>b</sup> | APE <sup>c</sup> | Pe <sup>b</sup> | APE <sup>c</sup> | Pe <sup>b</sup> | APE <sup>c</sup> | Pe <sup>b</sup> | APE <sup>c</sup> | Mean                              | Std. Dev.    | C.V. | Median |
| 1920                | 5.38            | 5.38             | 3.80            | 3.80             | 8.82            | 8.82             | 88.68           | 88.68            | 26.67                             | 41.39        | 1.55 | 7.10   |
| 1930                | 9.67            | 9.67             | 4.37            | 4.37             | -5.20           | 5.20             | -0.65           | 0.65             | 4.97                              | 3.70         | 0.74 | 4.79   |
| 1940                | -6.77           | 6.77             | -3.33           | 3.33             | 24.00           | 24.00            | 2.78            | 2.78             | 9.22                              | 10.01        | 1.09 | 5.05   |
| 1950                | -3.86           | 3.86             | 0.83            | 0.83             | -13.07          | 13.07            | -20.30          | 20.30            | 9.52                              | 8.88         | 0.93 | 8.47   |
| 1960                | -7.45           | 7.45             | -10.46          | 10.46            | -11.84          | 11.84            | 11.45           | 11.45            | 10.30                             | 1.99         | 0.19 | 10.96  |
| 1970                | -3.52           | 3.52             | 1.02            | 1.02             | 4.16            | 4.16             | 0.33            | 0.33             | 2.26                              | 1.87         | 0.83 | 2.27   |
| 1980                | 0.55            | 0.55             | 9.54            | 9.54             | 20.26           | 20.26            | 3.89            | 3.69             | 8.51                              | 8.67         | 1.02 | 6.62   |
| 1990                | -0.16           | 0.16             | 0.81            | 0.81             | -1.68           | 1.68             | 2.71            | 2.71             | 1.34                              | 1.11         | 0.82 | 1.25   |
| 2000                | -8.64           | 8.64             | -7.99           | 7.99             | -6.04           | 6.04             | -6.62           | 6.62             | 7.32                              | 1.20         | 0.16 | 7.31   |
| 2010                | 5.25            | 5.25             | 3.34            | 3.34             | 1.74            | 1.74             | 5.40            | 5.40             | 3.93                              | 1.74         | 0.44 | 4.30   |
| Mean                | -0.96           | 5.13             | 0.19            | 4.55             | 2.12            | 9.68             | 8.77            | 14.26            | APE (All states and target years) |              |      |        |
| Std. Deviation      | 6.19            | 3.18             | 5.98            | 3.58             | 12.55           | 7.64             | 29.33           | 26.82            | <b>Mean</b>                       | <b>8.40</b>  |      |        |
| Coeff. of Variation | -6.48           | 0.62             | 30.99           | 0.79             | 5.93            | 0.79             | 3.35            | 1.88             | <b>Std. Dev.</b>                  | <b>14.16</b> |      |        |
| Median              | -1.84           | 5.32             | 0.93            | 3.57             | 0.03            | 7.43             | 2.75            | 4.55             | <b>C. V.</b>                      | <b>1.69</b>  |      |        |
|                     |                 |                  |                 |                  |                 |                  |                 |                  | <b>Median</b>                     | <b>5.23</b>  |      |        |

<sup>a</sup> Total population = sum of the age group forecasts.

<sup>b</sup> Percent error.

<sup>c</sup> Absolute percent error.

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Table 4. Forecast Errors by Target Year, All Age Groups and States

| Target Year | MALPE <sup>a</sup> |           |        |        | MAPE <sup>b</sup> |           |
|-------------|--------------------|-----------|--------|--------|-------------------|-----------|
|             | Mean               | Std. Dev. | C.V    | Median | Mean              | Std. Dev. |
| 1920        | 22.63              | 36.41     | 1.61   | 5.48   | 23.57             | 35.74     |
| 1930        | 1.01               | 5.96      | 5.90   | 0.99   | 6.47              | 2.05      |
| 1940        | 2.88               | 13.06     | 4.53   | -0.73  | 11.15             | 7.27      |
| 1950        | -7.53              | 8.16      | -1.08  | -7.84  | 10.16             | 4.89      |
| 1960        | -2.91              | 9.63      | -3.31  | -6.89  | 9.76              | 3.52      |
| 1970        | 0.02               | 2.85      | 162.79 | -0.04  | 7.10              | 0.94      |
| 1980        | 7.29               | 8.94      | 1.23   | 5.14   | 10.33             | 7.76      |
| 1990        | 0.32               | 1.35      | 4.28   | 0.35   | 3.35              | 0.97      |
| 2000        | -6.86              | 1.04      | -0.15  | -6.79  | 6.86              | 1.04      |
| 2010        | 3.41               | 1.43      | 0.42   | 3.64   | 4.46              | 1.40      |

<sup>a</sup> MALPE = Mean Algebraic Percent Error.

<sup>b</sup> MAPE = Mean Absolute Percent Error.

<sup>c</sup> IOAE = Index of Allocation Error (Also known as the Index of Dissimilarity).

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Table 5. Forecast Errors by Age Group,  
All States and Target Years

| Age      | MALPE <sup>a</sup> | MAPE <sup>b</sup> | IOAE <sup>c</sup> |
|----------|--------------------|-------------------|-------------------|
| 0 to 4   | 6.35               | 23.61             | 33.01             |
| 5 to 9   | 5.66               | 22.61             | 31.94             |
| 10 to 14 | 1.09               | 6.85              | 6.75              |
| 15 to 19 | 1.67               | 7.32              | 6.88              |
| 20 to 24 | 4.73               | 11.34             | 7.70              |
| 25 to 29 | 4.64               | 13.05             | 9.80              |
| 30 to 34 | 2.25               | 11.79             | 9.65              |
| 35 to 39 | 1.92               | 8.97              | 6.18              |
| 40 to 44 | 1.19               | 7.23              | 5.60              |
| 45 to 49 | 0.15               | 6.15              | 5.15              |
| 50 to 54 | 0.81               | 4.85              | 5.03              |
| 55 to 59 | -0.26              | 4.92              | 4.73              |
| 60 to 64 | -0.74              | 4.80              | 4.23              |
| 65 to 69 | -0.24              | 5.07              | 3.40              |
| 70 to 74 | -1.84              | 5.39              | 2.87              |
| 75+      | -1.86              | 5.06              | 3.85              |

<sup>a</sup> MALPE = Mean Algebraic Percent Error.

<sup>b</sup> MAPE = Mean Absolute Percent Error.

<sup>c</sup> IOAE = Index of Allocation Error

## RESULTS

Turning to the results for all age groups and states combined summarized by target year, we can see in Table 4 that MALPE ranges from -7.53 in 1950 to 22.63 in 1920. The standard deviations for the MALPE scores range from 1.04 in 2000 to 36.41 in 1920 and the medians range from -6.79 in 2000 to 5.48 in 1920. In terms of MAPE, the means range from a low of 3.35 in 1990 to a high of 23.57 in 1920. The standard deviations for the MAPE scores range from a low of 0.94 in 1970 to a high of 35.74 in 1920.

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The median MAPE scores range from a low of 3.63 in 1990 to 11.12 in 1980. For the IOAE scores, the lowest mean (1.61) is found in 1990 and the highest in 1980 (7.63). The standard deviation of the IOAE scores range from a low of 0.29 in 1990 to a high of 1.50 in 1940. The median IOAE scores range from a low of 1.54 in 2010 to 7.83 in 1980.

## RESULTS

In terms of age group errors for all states and target years combined, the MALPEs in Table 5 suggest that bias is positive for younger age groups (0-4 through 50-54) and negative for older age groups (55-59 to 75+). Precision, as measured by MAPE, tends to improve as one moves from the youngest to the oldest age groups, as does allocation accuracy (IOAE). These generalizations are useful, but there are some important points that need more detailed discussion, especially in regard to precision (MAPE).

## RESULTS

For the two youngest age groups, 0-4 and 5-9, the MAPEs are very high at 23.61 and 22.61, respectively. The MAPEs for the next two age groups, 10-14 and 15-19, are much lower at 6.85 and 7.32, respectively. Precision decreases for the next two age groups 20-24 and 25-29, which have MAPEs of 11.34 and 13.05, respectively. After age 25-29 precision improves, going from a MAPE of 11.79 for age group 30-34 to a MAPE of 4.85 for age group 50-54, whereupon it tends to stabilize around 5.00% for all remaining age groups.



## DISCUSSION

Recalling the standards described in the section on “Measures of Error,” it appears that for the most part, the MALPE scores for the age forecasts indicate low levels of bias in that they are on average less than  $\pm 5\%$  across all states and years. The MAPE scores for the age forecasts indicate that they are moderately precise in that their average is more than 5%, but less than 10 percent across all state and years.

## DISCUSSION

In terms of allocation error, the average of the IOAE scores indicate that the Hamilton-Perry Method provides a close match to the census age distributions in that their average is less than 5% across all states and years. While there is a great deal of variation by state and year in the MALPE, MAPE and IOAE scores associated with the age group projections, we find that on the whole the Hamilton-Perry Method produces forecasts with low bias, high precision and low allocation error for the ten-year forecast horizons.

## DISCUSSION

As shown in Table 3, for the total population forecast for all states by target year, the lowest mean MAPE is found for the 1990 target year and the highest for 1920 (26.67). The lowest standard deviations for the MAPE scores are found in 1990 (1.11) and highest for 1920 (41.39). The lowest median for the MAPE is found in 1990 (1.25) and the highest in 1960 (10.96). Overall, the MAPE for the total population forecasts for all states and all years is 8.40 with a standard deviation of 14.16. The median MAPE score for the total population forecasts across all years and all states is 5.23. Overall, the method is moderately precise for the total population forecasts of all four states across all ten test points.

## DISCUSSION

In terms of the results by age group, it should not be surprising that the cohort change method is better able to capture older age groups, on average, than the very youngest since births are not part of a cohort change ratio. This applies to all three measures of accuracy, bias, precision, and allocation error. In addition, migration likely comes into play in that bias, precision, and allocation error decrease moving age groups 0-4 and 5-9 to age groups 10-14 and 15-19. Bias, precision, and allocation error all become worse moving to age groups 20-24 and 25-29 and 30-34, but then improve as one moves from these age groups to the remaining ones (35-39 to 75+).

## DISCUSSION

To some extent, migration also affects the forecast accuracy of the youngest two age groups (0-4 and 5-9) since they would be moving with their parents, who are likely to be in age groups 25-29, 30-34, and 35-39. Overall, we find that these effects are consistent with theory regarding migration in that those who tend to move are less socially integrated into communities than those who tend not to move and that community social integration tends to increase as one ages in regard to adults (Goldscheider 1978).

## DISCUSSION

In considering these results, two test target years (1940 and 1950) are directly related to major events that significantly affected demographic behaviors. The 1940 point encompasses the economic boom experienced in the 1920s and the economic depression during the 1930s and the large scale “baby bust” associated with it. The 1950 point encompasses the depression and baby bust period of the 1930s and the economic recovery stimulated by World War II and the initial part of the large scale “baby boom” from 1946 to 1950. These points, especially the latter one, are well-known in terms of being “unexpected” events. In terms of population forecasting, the task of capturing demographic change associated with them is very difficult.

## DISCUSSION

At this point, we suggest caution in using the Hamilton-Perry method beyond a ten-year forecast horizon. This is consistent with observations about the use of the Hamilton-Perry method in general (Smith, Tayman, and Swanson 2001; Swanson, Schlottmann, and Schmidt, 2010) and accuracy findings by Smith and Tayman (2003).<sup>4</sup> As such, this caution is not a major limitation. We also suggest that the Hamilton-Perry Method used with care when applied to small populations, such as those found at the county and sub-county levels.

## DISCUSSION

While our sample provides a wide range of demographic behavior in terms of size, age composition, and population changes, it is a sample of states, which means that greater variability in demographic characteristics found at sub-state levels is not present (Baker, Ruan, and Alcantara 2011, Swanson, Schlottmann, and Schmidt 2010). We suggest that further research using this approach would be useful by examining state level accuracy over forecast horizons longer than ten years and by examining the accuracy of smaller populations in conjunction over both longer forecast horizons and more cases, both from a geographic and a temporal standpoint.



**Questions?**

## REFERENCES CITED IN PRESENTATION

- Goldscheider, C. (1978). *Modernization, migration, and urbanization*. Paris, France: International Union for the Scientific Study of Population.
- Land, K. (1986). Methods for national population forecasts: A review. *Journal of the American Statistical Association* 81: 888-901.
- Smith, S.K. and Tayman, J. (2003). An evaluation of population projections by age. *Demography* 40 (4): 741-757.
- Smith, S.K., Tayman, J., and Swanson, D.A. 2001. *Population projections for state and local areas: Methodology and analysis*. New York, NY: Kluwer Academic/Plenum Press.
- Swanson, D.A. and Beck, D. (1994). A new short-term county population projection method. *Journal of Economic and Social Measurement* 20: 1-26.
- Swanson, D.A., Tayman, J., McKibben, J. and Cropper, M. (2012). A “blind” ex post facto evaluation of total population and total household forecast for small areas made by five vendors for 2010: Results by geography and error criteria. Presented at the 2012 Conference of the Canadian Population Society, Waterloo, Ontario, Canada.
- Swanson, D.A., Schlottmann, A., and Schmidt, R. (2010). Forecasting the population of census tracts by age and sex: An example of the Hamilton–Perry method in action. *Population Research and Policy Review* 29: 47-63.